

Synchrotron Self-Absorption Process in GRBs and the Isotropic Energy - Peak Energy Fundamental Relation

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Abstract

The existence of strong correlation between the peak luminosity (and/or bolometric energetics) of Gamma Ray Bursts (GRB) is one of the most intriguing problem of GRB physics. This correlation is not yet understood. Here we demonstrate that this correlation can be explained in framework of synchrotron self-absorption (SSA) mechanism of GRB prompt emission. We estimate the magnetic field strength of the central engine at the level $B \sim 10^{14} (10^3/\Gamma)^3 (1+z)^2$, where Γ is the Lorentz factor of fireball.

Key words: gamma ray bursts, synchrotron radiation, magnetic field.

1 Introduction

One of the recent important discoveries in gamma ray bursts (GRBs) is the discovery that the peak luminosity L_p (or/and the bolometric energetics) of GRBs correlates with their peak energy E_p (Amati et al., 2002; Amati, 2006; Yonetoku et al., 2004; Ghirlanda et al., 2004). This correlation allows to use it for constraining the cosmological parameters (see review Ghisellini (2007)). Liang et al. (2004) and Liang & Dai (2004) used the Amati relation for implications to fireball models.

The Amati relation between the isotropic-equivalent time integrated prompt energy E_{iso} and the peak energy \dot{E}_p in the comoving rest frame takes a form:

$$\dot{E}_{iso} \sim E_p^2 (1+z)^2 \quad (1)$$

Recently Li (2007) used the Amati relation as an example to test the cosmological evolution of GRBs. He used the Amati relation in logarithmic form (Amati, 2006):

$$\log E_{iso} = a + b \log E_p \quad (2)$$

where E_{iso} is the isotropic-equivalent energy defined in the $1 - 10^4$ KeV band in the GRB frame. Amati (2006) has obtained with a sample of 41 long GRBs the next values of parameters in (2): $a = -3.35$, $b = 1.75$ (E_p in KeV and E_{iso} in 10^{52} erg). These values have been obtained with the least squares method. If one uses the maximum likelihood method with an intrinsic dispersion the values of the numerical parameters are $a = -4.04$ and $b = 2.04$ (Amati, 2006; Li, 2007).

Yonetoku et al. (2004) have shown that also the peak luminosity $L_{p,iso}$ of the prompt emission correlates with E_p in the same way as E_{iso} :

$$E_p \sim L_p^{1/2}, \quad (L_p \equiv L_{p,iso}) \quad (3)$$

The scatter appears to be similar to the scatter of the Amati relation.

Ghirlanda et al. (2004) found that after correcting isotopic energetics by the collimation factor $E_\gamma = (1 - \cos \theta) E_{\gamma,iso}$, where θ is the collimation angle, the collimation corrected energy E_γ also correlated with E_p . The correlation is $E_p \sim E_\gamma^{0.7}$.

The next form of correlation has been obtained by Liang & Zhang (2005). Their correlation is entirely phenomenological and involves three observables and redshift. It is of the form $E_{iso} \sim E_p^2 t^{-1}$, where t

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is the scaled characteristic time of the jet development. If t is close to unit, then the Liang and Zhang correlation is entirely corresponding to the Ghirlanda correlation.

At last, the Firmani correlation (Firmani et al., 2006) links three quantities of the prompt emission: the peak bolometric and isometric luminosity L_p , the peak energy E_p and characteristic time $T_{0.45}$ which is time interval during that the prompt emission is above a certain level. This correlation takes a form: $L_p \sim E_p^{3/2} T_{0.45}^{-1/2}$, that is to be similar to the Ghirlanda relation.

Above mentioned relations show that their base is really the Yonetoku relation described by (3).

One of the interpretation of these relations has been recently suggested by Eichler & Levinson (2004) and was later developed by Levinson & Eichler (2005). They showed that both the slopes and scatters in the Amati and Ghirlanda relations, and the difference between them, are quantitatively consistent with their model, in which the $E_{iso} - E_p$ relation is due to viewing angle effect, i.e. consistent with pure geometrical effect.

In this paper we would like to show that the Amati relation can be received in framework of the synchrotron radiation mechanism with self-absorption. It means that the central engine of powerful GRB emitted in electromagnetic form $L_{iso} \sim 10^{52} - 10^{53}$ erg/s is tightly linked with a strong magnetic field. One cannot exclude that such powerful central engine appears in a result of the collapse of a star having substantial angular momentum. The collapse of a star may be accompanied by the formation of a quasi-static object - a spinar - whose equilibrium is maintained by centrifugal forces and a strong magnetic field (Ostriker, 1970; Lipunov, 1983; Lipunov & Gorbovsky, 2007).

2 Synchrotron Self-Absorption Process (SSA): Determination of Magnetic Field Strength

The magnetic field strength of cosmological GRB is believed to be estimated by assuming that the GRB energy spectrum is produced in a result of synchrotron emission with the observed radiation peak frequency ν_p at which the optical depth for synchrotron self-absorption is equal to unity being known.

Then averaged magnetic field strength is derived by the relation (Slysh, 1963; Hirotani, 2005):

$$B = 10^{-5} b(\alpha) \left(\frac{\nu_p}{1 \text{GHz}} \right)^5 \left(\frac{\phi}{0, "001} \right)^4 \left(\frac{1Jy}{S_p} \right)^2 \frac{\delta}{1+z} \quad (4)$$

The angular diameter - redshift relationship is derived as:

$$\phi = \frac{l(1+z)^2}{D_L} \quad (5)$$

where ϕ is an apparent angular diameter of the radiative source, l is the linear diameter of spherical source, and D_L is the luminosity distance, S_p is the peak flux density, δ is the boosting factor, the coefficient $b(\alpha)$ depends on the index of synchrotron power law spectrum. Its value lies in the region $b(\alpha) = 2 \div 3$, for the wide range of values α .

In our case one needs to take into account the effect of collimation and jet aperture. It means that the real linear diameter of GRB fireball is $l = l_\perp = \theta R$, where R is the jet length.

The peak flux density S_p can be presented in the form:

$$S_p = \frac{L_p}{4\pi D_L^2 \nu_p (1+z)} \quad (6)$$

where L_p is total luminosity and $L_p = L_{iso}$.

The peak energy E_p is equal to $E_p = h\nu_p$. The comoving peak energy is to be $\dot{E}_p = E_p(1+z)$.

3 The $L_{p,iso}$ - E_p relation in framework of SSA process

The maximum luminosity can be estimated in terms of the dissipation of magnetic energy:

$$L_p = L_{iso} \sim B^2 l^2 c \quad (7)$$

The magnetic field of the SSA process is derived by (4).

Using (4)-(6) we obtain:

$$L_p \sim \left[\left(\frac{\nu_p}{1\text{GHz}} \right)^7 (1+z)^9 \right]^2 l^{10} \delta^2 L_p^{-4} \quad (8)$$

Let us mention that the boosting factor δ is to be

$$\delta = \frac{1}{\gamma(1 - \beta \cos \theta)} \approx \frac{2}{\gamma \theta^2} \quad (9)$$

where θ is a jet angle, γ is the Lorentz factor and $\beta = v/c$. The comoving peak energy $\dot{E}_p \equiv h\nu_p(1+z) \sim \gamma$.

The next relation takes place:

$$\left(\frac{\nu_p}{1\text{GHz}} \right)^7 (1+z)^9 \sim (\dot{E}_p)^7 (1+z)^2 \quad (10)$$

Then the relation (8) transforms into:

$$L_{iso} \sim (\dot{E}_p)^{12} (1+z)^4 \theta^6 \frac{1}{L_{iso}^4} \quad (11)$$

and

$$L_{iso}^5 \sim E_p^{12} (1+z)^{16} \theta^6 \quad (12)$$

According to Sari et al. (1999) (see also Ghisellini (2007)):

$$\theta \sim \left(\frac{t_j}{1+z} \right)^{3/8} \left(\frac{n\eta_\gamma}{L_{iso}} \right)^{1/8} \sim \frac{1}{(1+z)^{3/8}} \frac{1}{L_{iso}^{1/8}} \quad (13)$$

This relation is valid on the case of relativistic jet propagation in a homogeneous circumburst medium, and what's more η_γ is the efficiency for converting the explosion energy to gamma rays, n is the density of the ambient medium and t_j is the time scale of jet evolution.

Using (12) and (13) one can get the next equation respect to $L_{iso} \equiv L_p$:

$$L_p^5 \sim E_p^{12} (1+z)^{55/4} \frac{1}{L_p^{3/4}} \quad (14)$$

or

$$L_p^{23/4} \sim E_p^{12} (1+z)^{55/4} \quad (15)$$

The solution of (14) and (15) gives:

$$L_p^{23/48} \sim E_p (1+z)^{55/48} \quad (16)$$

It means that we really obtain the Yonetoku correlation in framework of synchrotron self-absorption process, i.e. $E_p \sim L_p^{0.5}$. As $E_{iso} \approx L_p t_j$, the last relation is equivalent also to the Amati relation $E_p \sim E_{iso}^{0.5}$.

4 The Ghirlanda correlation

Ghirlanda et al. (2004) corrected the isotropic energetics by the factor $(1 - \cos \theta)$ and found that collimation corrected the Amati relation. Frail et al. (2001) found that the collimation clustered jet aperture angles into a narrow distribution producing a reservoir of explosion energy $E_\gamma = (1 - \cos \theta) E_{iso}$. From this relation for $\theta \ll 1$ the new correlation $E_p \sim E_\gamma^{0.7}$ is followed, if one uses (12) for the collimation angle θ . In this case instead of our (16) one can get the correlation

$$L_\gamma^{23/36} \sim E_p (1+z)^{55/48} \quad (17)$$

that is really approximately $E_p \sim L_\gamma^{0.7}$

5 The Estimation of Magnetic Field Strength of Central GRB Engine

Eq.(4) allows us to estimate the magnetic field strength of GRB central engine. We use for this estimation the results of spectral analysis of Swift long GRBs mode by Cabrera et al. (2007). They presented the best fit of correlation between \dot{E}_p and E_{iso} in the following logarithmic quantities:

$$\log \frac{\dot{E}_p}{1\text{KeV}} = (2.25 \pm 0.01) + (0.54 \pm 0.02) \log \frac{E_{iso}}{10^{52.44}\text{erg}} \quad (18)$$

Such correlation coincides with Amati relation (1) and was confirmed for 29 long GRBs observed by Swift satellite.

Using the relation: $E_{iso} = L_p t = 10^2 L_p t_2$, (4)-(6) and the following expression for the linear size of GRB central engine $l = \theta c t$, one can obtain the following relation instead of (4):

$$B = 1.2 \times 10^9 \left(\frac{\dot{E}_p}{1\text{KeV}} \right)^7 E_{iso,52}^{-2} (1+z)^2 t_2^6 \theta^4 \delta \quad (19)$$

From (9) it is followed the next expression for the boosting factor $\delta \sim \Gamma$, where Γ is the Lorentz factor of the GRB fireball. The typical relation between the collimation angle θ and the GRB Lorentz factor Γ is $\theta \sim 1/\Gamma$. Using also relation (18) we transform (19) into:

$$B = 1.2 \times 10^{14} (E_{iso,52})^{1.78} t_2^6 \Gamma_3^{-3} (1+z)^2 G \quad (20)$$

The typical Lorentz factor of GRB fireball is $\Gamma \sim 10^3 \Gamma_3$ (Dar & De Rujula, 2004; Dado et al., 2003). The estimation (20) gives for $z > 1$ the magnetic field strength as $B \geq 10^{14}$ G. This value is quite well corresponded to magnetar or/and spinar models of GRBs (Lipunov & Gorbovsky, 2007). If our suggestion on magnetar model is valid, it means that in our Galaxy four GRB events took place during its evolution time.

6 Summary

We demonstrate that the most important relation for physics of GRB between the peak luminosity (and the bolometric energetics) and their peak energy can be obtained in the framework of synchrotron mechanism of GRB emission taking into account the effect of self-absorption. In a result it is success to estimate the magnetic field strength of a central engine of GRB. Its value appears at the level of $> 10^{14}$ G.

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